

# Turbulent Cosmic-Ray Reacceleration at Radio Relics and Halos in Clusters of Galaxies

Yutaka Fujita

*Department of Earth and Space Science, Graduate School of Science, Osaka University,  
Toyonaka, Osaka 560-0043, Japan*

fujita@vega.ess.sci.osaka-u.ac.jp

Motokazu Takizawa

*Department of Physics, Yamagata University, 1-4-12 Kojirakawa-machi, Yamagata  
990-8560, Japan*

Ryo Yamazaki

*Department of Physics and Mathematics, Aoyama Gakuin University, Fuchinobe,  
Chuou-ku, Sagamihara 252-5258, Japan*

Hiroki Akamatsu

*SRON Netherlands Institute for Space Research, Sorbonnelaan 2, 3584 CA Utrecht, The  
Netherlands*

and

Hiroshi Ohno

*Tohoku Bunkyo College, 515 Katayachi, Yamagata 990-2316, Japan*

## ABSTRACT

Radio relics are synchrotron emission found on the periphery of galaxy clusters. From the position and the morphology, it is often believed that the relics are generated by cosmic-ray (CR) electrons accelerated at shocks through a diffusive shock acceleration (DSA) mechanism. However, some radio relics have harder spectra than the prediction of the standard DSA model. One example is observed in the cluster 1RXS J0603.3+4214, which is often called the “Toothbrush Cluster”. Interestingly, the position of the relic is shifted from that of a possible shock. In this study, we show that these discrepancies in the spectrum

and the position can be solved if turbulent (re)acceleration is very effective behind the shock. This means that for some relics turbulent reacceleration may be the main mechanism to produce high-energy electrons, contrary to the common belief that it is the DSA. Moreover, we show that for efficient reacceleration, the effective mean free path of the electrons has to be much smaller than their Coulomb mean free path. We also study the merging cluster 1E 0657–56 or the “Bullet Cluster”, in which a radio relic has not been found at the position of the prominent shock ahead of the bullet. We indicate that a possible relic at the shock is obscured by the observed large radio halo that is generated by strong turbulence behind the shock. We propose a simple explanation if the morphological differences of radio emission among the Toothbrush, the Bullet, and the Sausage (CIZA J2242.8+5301) Clusters.

*Subject headings:* cosmic rays — shock waves — turbulence — galaxies: clusters: individual (1RXS J0603.3+4214, 1E 0657–56, CIZA J2242.8+5301)

## 1. Introduction

Radio relics and halos are diffuse sources that are often found in galaxy clusters. While the former are located on the periphery of a cluster, the latter are observed at the center of a cluster. They both seem to be related to an ongoing merger event (Ferrari et al. 2008). Since they are synchrotron emission, their presence indicates that cosmic-ray (CR) electrons are widely distributed in clusters. Observationally, the relics appear to be associated with shocks created by cluster mergers (Giacintucci et al. 2008; Macario et al. 2011; Bourdin et al. 2013; Ogreaan & Brüggen 2013), and thus diffusive shock acceleration (DSA) may be working there (Roettiger et al. 1999; Fujita & Sarazin 2001; Kang et al. 2012; Yamazaki & Loeb 2015). On the other hand, the halos may originate from electrons (re)accelerated by cluster-wide turbulence excited by cluster mergers (Schlickeiser et al. 1987; Brunetti et al. 2001; Petrosian 2001; Fujita et al. 2003). They may also be produced by electrons created through hadronic interactions (Miniati et al. 2001; Pfrommer & Enßlin 2004; Keshet & Loeb 2010; Enßlin et al. 2011).

The galaxy cluster 1RXS J0603.3+4214 has a famous radio relic called “toothbrush” (van Weeren et al. 2012). Thus, we call this cluster the “Toothbrush Cluster” from now on. The relic is located  $\sim 1$  Mpc away from the cluster center. The Mach number of a possible shock associated with the relic is less than 2 (Ogreaan et al. 2013; Itahana et al. 2015; van Weeren et al. 2015). Curiously, the observed radio spectrum is much harder than the prediction of the standard DSA model based on the Mach number. Moreover, the position

of the relic does not seem to coincide with that of the shock (Ogrean et al. 2013). These facts suggest that the CR electrons responsible for the relic are not accelerated through the DSA mechanism, although projection effects of multiple shocks may account for them (?; see also Vazza et al. 2012; Skillman et al. 2013).

The merging cluster 1E 0657–56 is known as the “Bullet Cluster”, because a small cluster (“bullet”) is passing through a large cluster with a velocity of  $\gtrsim 4000 \text{ km s}^{-1}$  (Markevitch et al. 2002; Markevitch 2006). X-ray observations show that this cluster has a prominent shock with a Mach number of  $\mathcal{M} = 3$  ahead of the bullet. Behind the shock, a large radio halo is developing (Liang et al. 2000; Markevitch et al. 2002; Shimwell et al. 2014). However, despite of the relatively large Mach number, a radio relic has not been found at the position of the shock in this cluster. This seems to contradict the fact that  $\mathcal{M} \sim 3$  is a normal value for the development of a relic at a shock (Akamatsu & Kawahara 2013).

In this paper, we explore the possibility that the relic in the Toothbrush Cluster is mainly associated with turbulence developed *behind* a shock. This may contradict the common idea that radio relics are basically generated by the DSA and radio halos are generated by the turbulent reacceleration. In our model, CR electrons weakly accelerated at the shock are significantly reaccelerated in the turbulence (Markevitch et al. 2005; Mertsch & Sarkar 2011; ?; see also Inoue et al. 2009). In other words, the DSA at the shock only provides seed CR electrons that are reaccelerated in the turbulence. Since the shock is weak, the radio emission at the shock is not strong. However, if the turbulent reacceleration is effective enough, the radio emission from the reaccelerated electrons can be strong, and the spectrum can be harder than the prediction of the DSA model. Moreover, the difference of the position of the shock and the relic in the Toothbrush Cluster can naturally be explained. We also consider the Bullet Cluster as an extreme case of our model. That is, the reacceleration is so effective that the emission from the reaccelerated electrons, which is observed as a radio halo, obscures a possible radio relic at the shock.

## 2. Models

We assume that electrons accelerated at a shock are reaccelerated by turbulence in the downstream of the shock. For a given upstream gas density  $\rho_u$ , an upstream velocity  $V_u$  on the shock frame, and a Mach number  $\mathcal{M}$ , the downstream density and the velocity are  $\rho_d = r_c \rho_u$  and  $V_d = V_u / r_c$ , respectively, where

$$r_c = \frac{(\gamma_g + 1)\mathcal{M}^2}{(\gamma_g - 1)\mathcal{M}^2 + 2} \quad (1)$$

is the shock compression ratio and  $\gamma_g = 5/3$  is the adiabatic index. For a given temperature  $T_u$  of the upstream, that of the downstream is written as

$$\frac{T_d}{T_u} = \frac{[2\gamma_g\mathcal{M}^2 - (\gamma_g - 1)][(\gamma_g - 1)\mathcal{M}^2 + 2]}{(\gamma_g + 1)^2\mathcal{M}^2}. \quad (2)$$

We assume a planar shock.

The momentum spectrum of electrons accelerated through the DSA mechanism at the shock is

$$n_0(p) = A_0 p^{-s}, \quad (3)$$

where

$$s = \frac{r_c + 2}{r_c - 1} \quad (4)$$

(Blandford & Eichler 1987). Here the number density of electrons with momenta between  $p$  and  $p + dp$  is  $n_0(p)dp$ . The lowest momentum of CR electrons is  $p_{\min} = m_e c$ . The normalization  $A_0$  is given as follows. The kinetic energy flux of gas from the upstream of the shock is  $\rho_u V_u^3/2$ . We assume that the fraction  $\chi_e$  of that flux becomes the kinetic energy of CR electrons. Thus, the energy density of CR electrons just downside of the shock is  $\epsilon_{e,sh} = \chi_e \rho_u V_u^3/(2V_d) = \chi_e \rho_u V_u^2 r_c/2$ . On the other hand,

$$\epsilon_{e,sh} = \int_{p_{\min}}^{\infty} m_e c^2 (\gamma - 1) n_0(p) dp, \quad (5)$$

where  $\gamma$  is the Lorentz factor for the particles. Thus,

$$A_0 = \frac{\chi_e \rho_u V_u^3 r_c}{2m_e c^2} / \int_{p_{\min}}^{\infty} (\gamma - 1) p^{-s} dp. \quad (6)$$

In the downstream of the shock, electrons are reaccelerated in turbulence and the spectrum changes as a function of time,  $n(t, p)$ , where  $n(t = 0, p) = n_0(p)$ . The evolution is dictated by the Fokker-Planck equation,

$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial p} \left( p^2 D_{pp} \frac{\partial n}{\partial p} \frac{1}{p^2} \right) + \frac{\partial}{\partial p} \left( \frac{dp}{dt} n \right) = 0, \quad (7)$$

where  $D_{pp}$  is the diffusion coefficient in momentum for scattering. Since CR electrons are swept downstream with thermal gas, the evolution of the spectrum described by equation (7) can be interpreted as the spatial change of the spectrum. The distance from the shock  $x$  is represented by the time  $t$  as in  $x = V_d t$ . The value of  $x$  increases downstream.

We consider resonant acceleration in which Alfvén waves scatter particles because Alfvénic turbulence may cascade to a very small scale (see Section 2.2.2 of Brunetti & Jones 2014),

which reduces an effective mean free path of particles and increases efficiency of reacceleration as is shown below. The momentum diffusion coefficient is

$$D_{pp} \sim \frac{1}{9} p^2 \frac{v_A^2}{D_{xx}}, \quad (8)$$

where  $v_A$  is the Alfvén velocity and  $D_{xx}$  is the spatial diffusion coefficient given by

$$D_{xx} \sim \frac{cl_{\text{mfp}}}{3}, \quad (9)$$

where  $l_{\text{mfp}}$  is the electron mean free path (Ohno et al. 2002; see also Isenberg 1987; Schlickeiser 1989; Fujita et al. 2003; Brunetti et al. 2004). In the case of turbulent acceleration by magnetosonic waves, it is often assumed that  $l_{\text{mfp}}$  is the Coulomb mean free path of thermal particles  $l_{\text{mfp,C}}$  (e.g. ZuHone et al. 2013; see also Brunetti & Lazarian 2007), which depends on  $\rho_d$  and  $T_d$ . In this study, we treat  $l_{\text{mfp}}$  as a free parameter represented by

$$l_{\text{mfp}}(t, p) = \eta(t)(p/p_0)^{2-q} l_{\text{mfp,C}}, \quad (10)$$

where  $\eta(t)$  is the reduction factor ( $\eta \leq 1$ ),  $p_0$  is the reference momentum, and  $q$  is the parameter that represents the property of the turbulence. We fix  $p_0$  at  $10^4 m_e c$  because electrons around that momentum are responsible for the observed synchrotron emissions. We assume that  $q = 5/3$ , which represents the Kolmogorov case. Equation (10) means that for  $p \sim p_0$  and  $\eta \ll 1$  the turbulent cascade extends to scales much smaller than  $l_{\text{mfp,C}}$  or waves are created by plasma instabilities at the small scales (Brunetti & Lazarian 2011; Brunetti & Jones 2014). We consider time (or spatial) dependence of  $\eta$  because the waves responsible for the particle scattering develop at  $x \lesssim L_t$  according to the cascade of the eddies, where  $L_t$  is the scale comparable to the size of the largest eddies. Since the evolution of the turbulence should reflect this scale,  $\eta(t)$  evolves on a timescale of  $\sim L_t/V_d$ , which is much larger than the scattering time of a particle  $l_{\text{mfp}}/c$ . In this study, we do not discuss the microphysics of the turbulent reacceleration in detail because it is not well known (?). Instead, we treat it phenomenologically and obtain the value and changing rate of  $\eta(t)$  that are consistent with observations of radio profiles. Moreover, we do not consider the back reaction of CRs on the turbulence for simplicity because we do not specify the turbulence on the microscale; the effect is included in  $\eta(t)$  instead. In equation (7),  $dp/dt$  represents cooling of electrons. We include synchrotron, inverse Compton scattering, nonthermal bremsstrahlung, and Coulomb interaction. The synchrotron emission depends on the magnetic field of the downstream of the shock  $B_d$ .

### 3. Results

#### 3.1. Toothbrush Cluster

We assume that  $V_u = 2500 \text{ km s}^{-1}$ ,  $\rho_u = 5.7 \times 10^{-28} \text{ g cm}^{-3}$ ,  $\mathcal{M} = 1.5$ , and  $T_u = 4 \text{ keV}$  based on the X-ray observations (Ogrea et al. 2013; Itahana et al. 2015). Moreover, we assume that  $L_t = 90 \text{ kpc}$  and  $\chi_e = 2.5 \times 10^{-7}$ . These are chosen to be consistent with radio observations (see later). For these values, the Coulomb mean free path in the downstream is  $l_{\text{mfp,C}} \sim 19 \text{ kpc}$ . The magnetic field in the downstream is  $B_d = 2.0 \mu\text{G}$ , which is given by observations (Itahana et al. 2015).

The functional form of the reduction factor is

$$\eta(t)(p/p_0)^{2-q} = \min\{\eta_{\min}(p/p_0)^{2-q} \exp(t/t_0), 1\}. \quad (11)$$

In the fiducial model for the Toothbrush Cluster, we assume that  $\eta_{\min} = 1.3 \times 10^{-4}$ , and  $t_0 = L_t/V_d$ . This change of the mean free path is associated with the decay of turbulence. We assume that the Alfvén velocity behind the shock as  $v_A = B_d/\sqrt{4\pi\rho_d}$ . For the parameters we adopted, the pressure of the magnetic fields just behind of the shock,  $B_d^2/(8\pi)$ , is smaller than the thermal pressure there. Note that the parameters  $\eta_{\min}$  and  $B_d$  are degenerated for reacceleration because both smaller  $\eta_{\min}$  and larger  $B_d$  result in higher efficiency of reacceleration (equations 8 and 9), although  $B_d$  affects synchrotron cooling. The parameters  $L_t$  and  $\chi_e$  determine the size of the turbulent region and the normalization of the CR spectrum, respectively. The free parameters in this model are basically  $\eta$  (or  $\eta_{\min}$  and  $L_t$ ) and  $\chi_e$ .

Figure 1a shows the evolution of the electron momentum spectrum. Initially, the spectrum is soft because of the low Mach number ( $\mathcal{M} = 1.5$ ). In the strong turbulence behind the shock, the electrons are reaccelerated, which continues until  $t \lesssim t_0 = 60 \text{ Myr}$ . At this stage, the electron spectrum becomes harder. After that, the reacceleration weakens owing to the decay of the turbulence and radiative cooling dominates the reacceleration. As a result, the number of high-energy electrons ( $\gamma \gtrsim 10^4$ ) decreases. Figure 1b presents the integrated synchrotron flux of the relic. We assume that the relic as a whole is stationary and both the relic’s width and the depth along the line of sight are 500 kpc. The spectrum is not represented by a single power law and is broadly consistent with the observation (?).

Figure 1c shows the synchrotron surface brightness at 1382 MHz as the function of the distance from the shock  $x$ . We assume that the shock normal is perpendicular to the line of sight and that the depth of the radio relic along the line of sight is 500 kpc. For comparison, we show the observation of the west part of the Toothbrush at the same frequency (van Weeren et al. 2012). The position of the shock is set at the position indicated by the X-

ray observations (Ogreaan et al. 2013). Our model can reproduce the observed sharp increase of the brightness behind the shock ( $x \sim 140$  kpc). The peak of the position is determined by the acceleration time,  $t_{\text{acc}} \sim p^2/D_{pp}$ . At  $x = 0$  and  $p = p_0$ , the timescale is  $t_{\text{acc}} = 50$  Myr, and the spatial scale is  $x_{\text{acc}}(x = 0) = V_d t_{\text{acc}}(x = 0) \sim 74$  kpc. Since  $t_{\text{acc}}$  increases as  $x$  increases and  $D_{pp}$  decreases, the position of the peak ( $x \sim 140$  kpc) is a factor of two larger than  $x_{\text{acc}}(x = 0)$ . We note that the position of the peak is closer to the shock compared with the observations ( $x \sim 300\text{--}400$  kpc; Ogreaan et al. 2013). This may indicate that it takes some time for the turbulence to develop behind the shock and that the reacceleration is delayed. Or there may be an error for the position of the weak shock. In fact, recent *Chandra* observations suggest that the shock is located just at the northern edge of the relic but not at the peak (van Weeren et al. 2015), which is consistent with Figure 1c because the observed radio profile (dotted line) should be shifted to the left by  $\sim 150\text{--}200$  kpc. In Figure 1c, we also plot the spectral index of the radio emission, which is measured between 147 and 2272 MHz. At  $x = 0$  ( $t = 0$ ), the index is predicted by the standard DSA model,

$$\alpha_{\text{DSA}} = \frac{1}{2} - \frac{\mathcal{M}^2 + 1}{\mathcal{M}^2 - 1} \quad (12)$$

(e.g. Blandford & Eichler 1987), and  $\alpha_{\text{DSA}} = -2.1$  here. The index increases up to  $\alpha \sim -0.6$  at the peak of the surface brightness, which is consistent with the observations (Fig. 8 in van Weeren et al. 2012). Note that the rapid decrease of the index just behind the shock is caused by radiative cooling before reacceleration becomes effective. On the other hand, the diffuse emission observed at  $x \gtrsim 700$  kpc cannot be reproduced by our model. This suggests that the turbulence does not completely decay, and weak reacceleration continues there. We note that polarization has been detected at the relic of the Toothbrush Cluster (van Weeren et al. 2012). This may be related to compression of large-scale magnetic fields (Iapichino & Brüggen 2012).

Figure 2 shows the results when  $\eta_{\text{min}} = 5.3 \times 10^{-4}$  and  $\chi_e = 0.01$ ; the other parameters are the same as the fiducial model. The larger  $\eta_{\text{min}}$  means that the turbulent reacceleration is less effective than that in the fiducial model. We compensate the inefficiency with the larger  $\chi_e$ . Although this model can reproduce the surface brightness at the peak, the spectral index is much smaller than the observations ( $\alpha \sim -0.6$ ; van Weeren et al. 2012) because of the weaker reacceleration. These show that the reduction factor must be small enough to explain both observations of surface brightness and spectral index.

### 3.2. Bullet Cluster

The fiducial parameters for the Bullet Cluster are  $V_u = 4700 \text{ km s}^{-1}$ ,  $\rho_u = 1.1 \times 10^{-27} \text{ g cm}^{-3}$ ,  $\mathcal{M} = 3.0$ , and  $T_u = 9.2 \text{ keV}$ , which are based on the X-ray observations (Markevitch 2006). Other fiducial parameters are  $B_d = 30 \mu\text{G}$ , and  $\chi_e = 3 \times 10^{-10}$ . These parameters are chosen so that the results are consistent with radio observations (see below). The magnetic pressure is smaller than the thermal pressure. Since the radio emission covers the whole cluster (Liang et al. 2000; Shimwell et al. 2014), we expect that turbulence is developing in the whole cluster and the decay scale is much larger than that of the Toothbrush Cluster. Thus, we simply assume that the reduction factor is constant ( $L_t \rightarrow \infty$ ), and it is  $\eta = 1 \times 10^{-3}$ . The Coulomb mean free path in the downstream is  $l_{\text{mfp},C} \sim 190 \text{ kpc}$ .

Figure 3a shows the evolution of the electron momentum spectrum. Compared to Figure 1a, the initial spectrum is harder because of the larger Mach number ( $\mathcal{M} = 3.0$ ). As the electrons are reaccelerated, the peak at  $p/(m_e c) \sim 10^3\text{--}10^4$  becomes prominent. Although radiative cooling gradually becomes effective, turbulent acceleration compensates the effect. For  $t \gtrsim 50 \text{ Myr}$ , the spectrum does not much change because the acceleration balances with the cooling. Figure 3b shows the synchrotron spectrum of the whole cluster, which is not represented by a power law because it reflects the shape of the momentum spectrum (Fig. 3a). We assume that the radio halo’s length is  $x = 1.6 \text{ Mpc}$ . Both the width and the depth along the line of sight are  $500 \text{ kpc}$ . The normalization of the synchrotron spectrum depends on the assumed geometry of the radio-emitting region. Figure 3c shows the profile of surface brightness as a function of the distance from the shock. We simply assume that the position of the shock is at the tip of the observed radio emission (Fig. 5 of Shimwell et al. 2014). Although our prediction does not perfectly reproduce the observed profiles, it reproduces the general trend, such as a steep rise behind the shock and a fairly flat profile after that. We note that the actual cluster is very complicated. For example, there is a bullet or a sharp X-ray peak at  $x = 180 \text{ kpc}$  (Shimwell et al. 2014), and the turbulence is not expected to be uniform behind the shock. Thus, it would be useless to fine-tune parameters in our simple model in order to perfectly reproduce the observations. The spectral index at  $x = 0$  is  $\alpha_{\text{DSA}} = -0.75$  (equation 12). For  $x \gtrsim 100 \text{ kpc}$ , the index is almost constant (Fig. 3c). The average index ( $\alpha \sim -1.5$ ) is consistent with the observations (Shimwell et al. 2014).

Our results indicate that the surface brightness starts to rise at  $x \sim 30 \text{ kpc}$  (Fig. 3c), and the gap between the shock and the rising point is smaller than the resolution of the current radio telescope ( $\sim 100 \text{ kpc}$ ; Shimwell et al. 2014). At  $x = 0$  and  $p = p_0$ , the acceleration time and the spacial scale are  $t_{\text{acc}} = 59 \text{ Myr}$  and  $x_{\text{acc}} = 95 \text{ kpc}$ , respectively, which are comparable to those for the Toothbrush Cluster. Observations with a higher resolution in the future are especially important to reveal the turbulent reacceleration just behind the



shock because the rising point reflects the efficiency of the reacceleration that depends on parameters such as  $\eta$  and  $v_A (\propto B_d)$ . The gap is smaller and the spectrum is harder if  $\eta$  is smaller and/or if  $v_A$  is larger because acceleration is more efficient.

The Mach number of the shock observed in the Bullet Cluster is relatively large ( $\mathcal{M} \sim 3$ ; Markevitch 2006), and it is comparable to that of the shock observed in CIZA J2242.8+5301 (Akamatsu & Kawahara 2013; ?), at which a sharp radio relic called “sausage” has been observed (van Weeren et al. 2010). In contrast with CIZA J2242.8+5301 (the Sausage Cluster hereafter), however, no bow-like radio structure featuring a relic has been observed in the Bullet Cluster. This means that a Mach number is not the only factor that determines radio morphology. In order to find the reason, we try to reproduce the sausage-like structure with minimum changes of the parameters from the fiducial model. Figure 4 shows the profile of the surface brightness when  $\eta = 1$  and  $\chi_e = 3 \times 10^{-6}$ ; other parameters are the same as in the fiducial model for the Bullet. The larger values of  $\eta$  and  $\chi_e$  mean that while the turbulent reacceleration is less efficient, the shock acceleration is more efficient. In Figure 4, the radio emission peaks at the shock ( $x = 0$ ) and rapidly decreases at  $x > 0$ , which are the properties of the DSA model and are similar to the radio relic in the Sausage Cluster (van Weeren et al. 2010), although the parameters are not chosen to precisely reproduce the Sausage Cluster. This suggests that the difference between the Bullet Cluster and the Sausage Cluster reflects the difference of contribution between the reacceleration in the turbulence, which depends on  $\eta$ , and the acceleration at the shock, which depends on  $\chi_e$ . In other words, the results for the fiducial parameters (Fig. 3) indicate that the emission from the radio halo obscures that from the relic at the shock.

## 4. Discussion

What makes the differences of radio emissions among the Toothbrush Cluster, the Bullet Cluster, and the Sausage Cluster? The results in section 3 show that  $\eta$  and  $\chi_e$  can be the key parameters to explain them together; the former reflects the reacceleration efficiency in the turbulence and the latter reflects the DSA efficiency at the shock. We speculate that there are three elements that determine  $\eta$  and  $\chi_e$ ; development of turbulence, the Mach number of shocks, and preexisting CRs.

In our study, the difference of radio profiles between the Toothbrush Cluster (Fig. 1c) and the Bullet Cluster (Fig. 3c) is mainly caused by the difference of  $\eta(t)$ , which reflects the decay of turbulence. One possible reason may be disruption of a cool core. If a fast-moving core is being violently destroyed through interaction with the ambient medium, strong turbulence may develop. This may be the case of the Bullet Cluster (Markevitch

2006). On the other hand, the Toothbrush Cluster does not seem to have a breaking cool core (Ogrea et al. 2013; van Weeren et al. 2015). This may be one reason that strong turbulence is not developing in the central region. The mass ratio of subcluster components in a merging cluster may be a clue. For the Bullet Cluster, the ratio is expected to be around 1 : 0.1 (Takizawa 2006; Akahori & Yoshikawa 2012). Weak-lensing analysis showed that the gas component of the smaller mass component (bullet) is decoupled from its dark matter component, and the shape of the shock front appears to be irrelevant to the dark matter distribution (Clowe et al. 2004). Thus, the gravitational potential of the smaller component could not hold its gas and cool core. The Toothbrush Cluster may be formed through mergers of three clusters with the mass ratio of 1 : 1 : 0.07 (Brüggen et al. 2012; see also Jee et al. 2015). If their mergers are asymmetric, gas motion in the cluster may be complicated and possible strong turbulence associated with the smallest component may not develop at the cluster center. At the present, only weak turbulence may remain at the center, and the radio emission from CRs accelerated by the turbulence may be observed as a diffuse halo at  $x \gtrsim 700$  kpc (Fig. 1c). For the Sausage Cluster, Okabe et al. (2015) recently showed that the mass ratio of two components of the cluster is 1 : 0.5 and that the curved shapes of the two radio relics well match the density contours of the dark matter. That is, the gas distribution fairly follows the dark matter distribution. These may indicate that the mass similarity of the two components may prevent one-sided destruction of the smaller component and development of strong turbulence. Of course, if the mass ratio is too large, the smaller component hardly affects the main cluster. Thus, a moderate mass ratio may be preferable for development of turbulence.

Our study suggests that the relic of the Toothbrush Cluster is not basically a result of the DSA, in contrast with the Sausage Cluster. Their different Mach numbers might be responsible for that. For example, Ryu et al. (2003) indicated that CR acceleration efficiency drops at  $\mathcal{M} \lesssim 3$ . Thus, the relic of the Toothbrush Cluster ( $\mathcal{M} \sim 1.5$ ) may not satisfy a necessary condition of effective DSA acceleration ( $\mathcal{M} \gtrsim 3$ ). On the other hand, the Mach numbers for the Bullet Cluster and the Sausage Cluster satisfy this condition. Their difference of radio morphology may reflect existence or nonexistence of seed CR electrons *ahead* of the shocks (Ensslin et al. 1998; Kang et al. 2012; Pinzke et al. 2013). The normalization of the CR spectrum at the shock,  $\chi_e$ , for Figure 4 is  $10^4$  times larger than that for Figure 3. This may mean that effective DSA acceleration is required for a relic to be bright at the shock like the Sausage Cluster (Fig. 4). The preexisting CR electrons may enhance the efficiency (Kang et al. 2012; Pinzke et al. 2013). On the other hand, if there are few preexisting CR electrons, the efficiency is small and the radio emission is dim at the shock. However, if strong turbulence is developed behind the shock, even the electrons with small energies are effectively reaccelerated and may form a radio halo (Fig. 3).

## 5. Summary

We have investigated the origin of radio relics on the periphery of galaxy clusters. We focus on the turbulent reacceleration of CR electrons that had been weakly accelerated at a shock. We considered Alfvén waves as scatterers of the CR electrons. We found that the effective mean free path of the particles must be much smaller than the Coulomb mean free path for efficient reacceleration. This may be realized if the turbulence cascades to the smaller scale or plasma instabilities excite waves at that small scale. Using our model, we reproduced the hard spectrum of the relic observed in the cluster 1RXS J0603.3+4214 (the Toothbrush Cluster), which cannot be explained by the standard DSA model. In our model, the position of the relic does not coincide with that of the shock. The difference of the positions has actually been observed in the Toothbrush Cluster. Our results show that for some relics turbulent reacceleration may be the main mechanism to produce high-energy electrons, although the turbulent reacceleration is generally associated with giant radio halos. We also applied our model to the merging cluster 1E 0657–56 (the Bullet Cluster), in which a radio relic has not been found in spite of the large Mach number of the bow shock. We showed that strong turbulence behind the shock can reaccelerate CR electrons. The bright synchrotron emission from those electrons is observed as a radio halo and it obscures a possible radio relic at the shock. If the acceleration of the shock were more efficient and the reacceleration in the turbulence were less efficient, a sharp radio relic should have been observed in the Bullet Cluster as in CIZA 2242.8+5301 (the Sausage Cluster). Our results suggest that various radio relics and halos in clusters can be explained all together by only two factors: the reacceleration efficiency in the turbulence and the DSA efficiency at the shock. We speculate that development of turbulence, the Mach number of shocks, and preexisting CRs determine those two factors.

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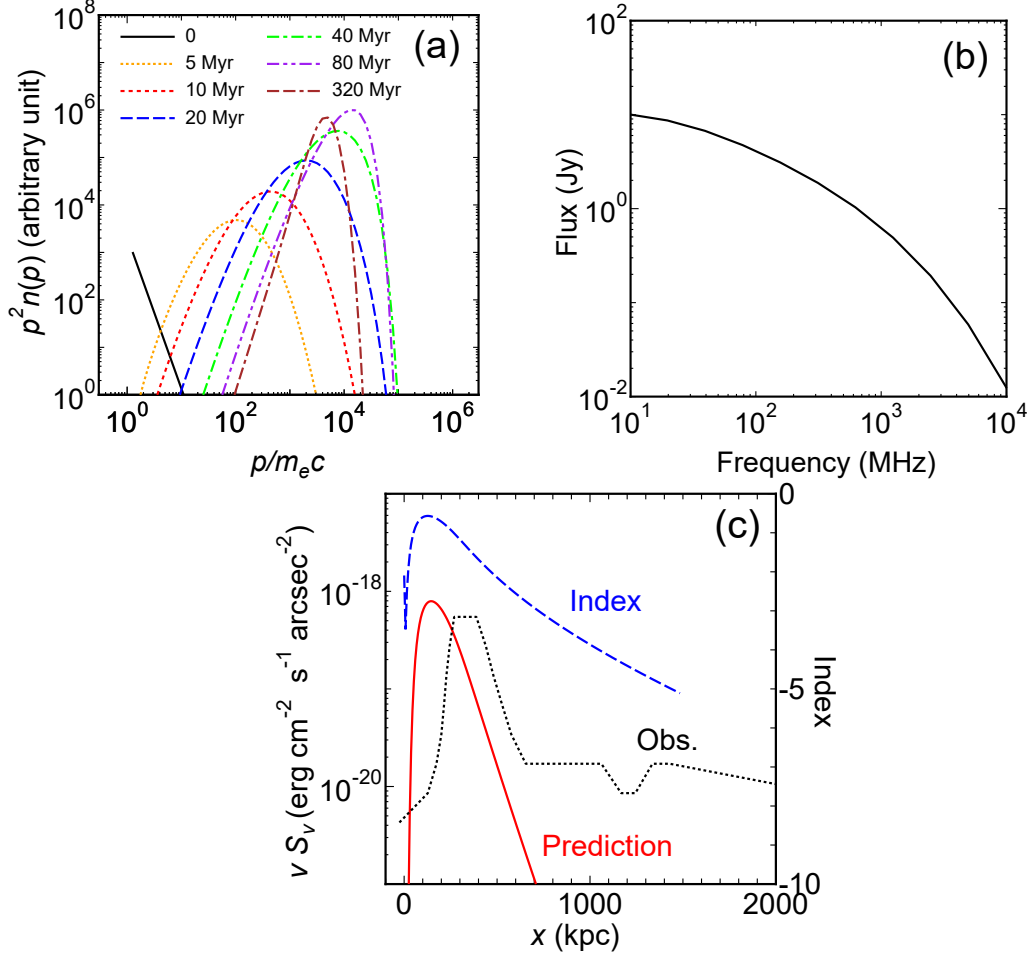


Fig. 1.— Results for the Toothbrush Cluster using the fiducial parameters. (a) Electron spectra behind the shock. Time  $t$  is shown in the legends;  $t = 320$  Myr corresponds to the distance of  $x = 480$  kpc. (b) Integrated radio spectrum. (c) Synchrotron surface brightness at 1382 MHz as the function of the distance from the shock. The solid curve is our prediction, and the dotted curve is the observation (van Weeren et al. 2012). The dashed curve is our prediction for the spectral index between 150 and 2272 MHz.

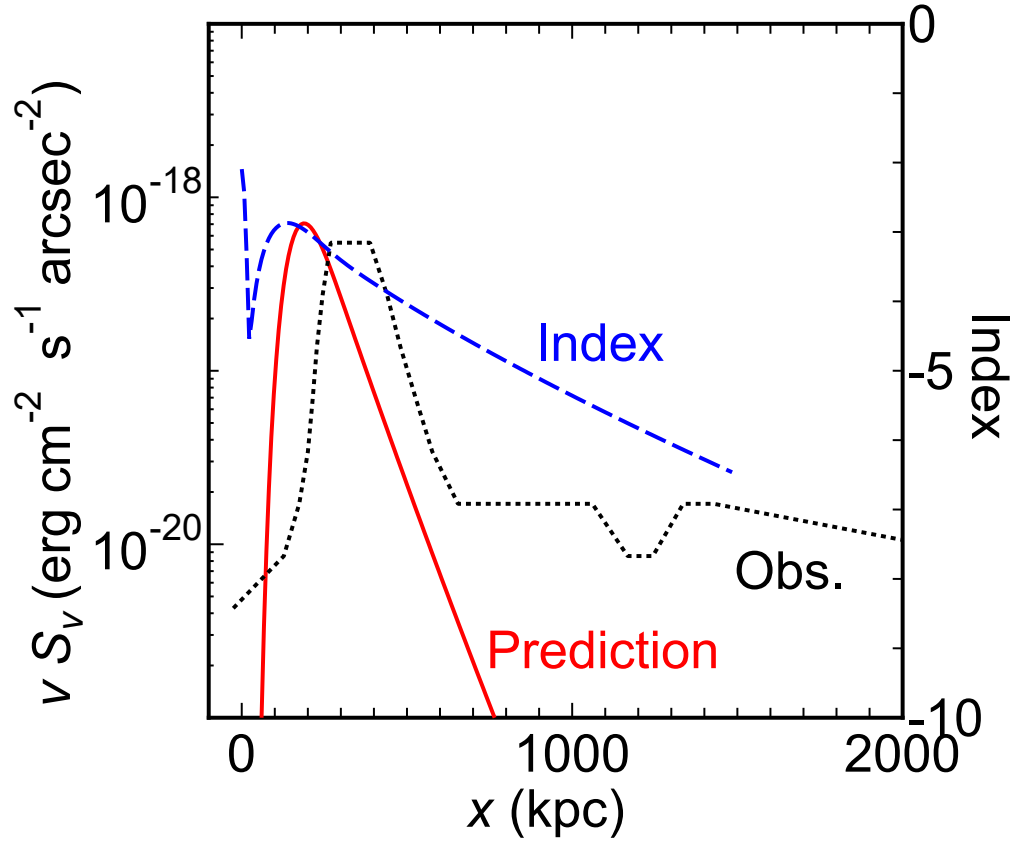


Fig. 2.— Same as Figure 1(c) but for  $\eta_{\min} = 5.3 \times 10^{-4}$  and  $\chi_e = 0.01$ .



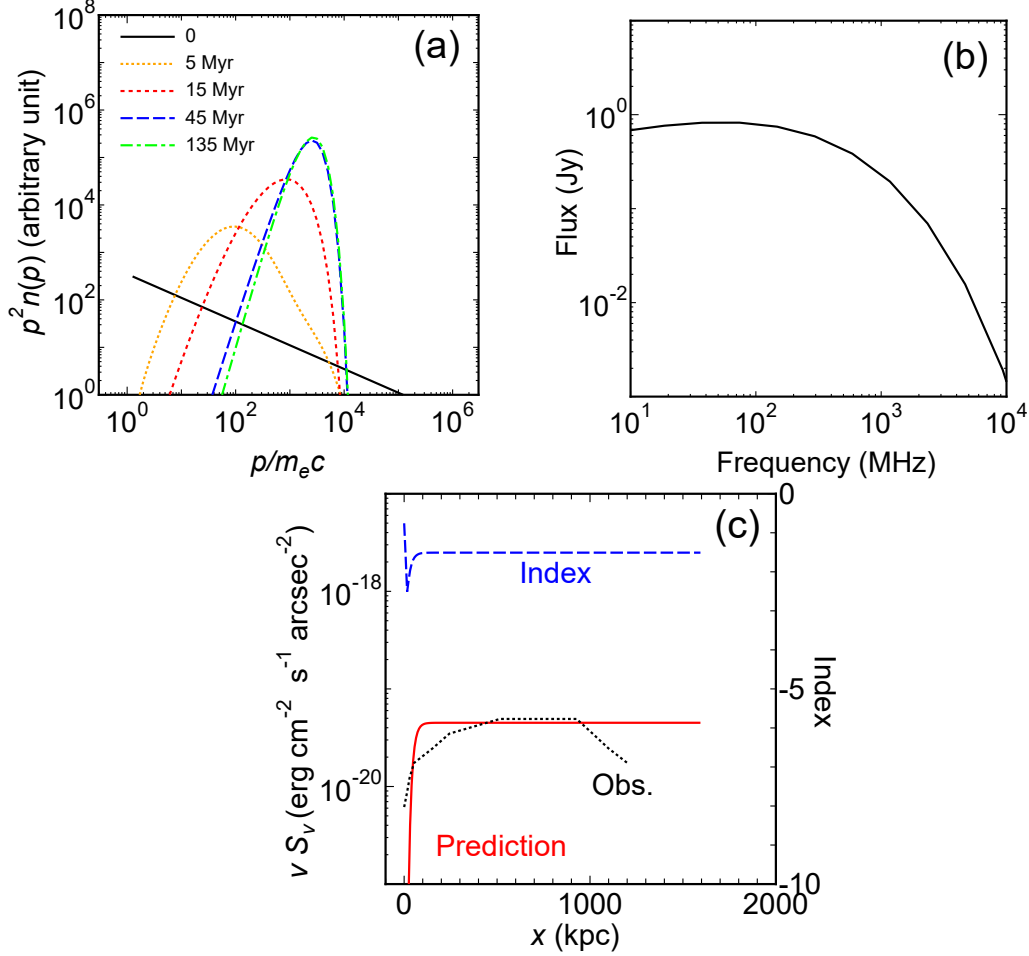


Fig. 3.— Results for the Bullet Cluster using the fiducial parameters. (a) Electron spectra behind the shock. Time  $t$  is shown in the legends;  $t = 135$  Myr corresponds to the distance of  $x = 216$  kpc. (b) Integrated radio spectrum. (c) Synchrotron surface brightness at 2.1 GHz as a function of the distance from the shock. The solid curve is our prediction and the dotted curve is the observation (Shimwell et al. 2014). The dashed curve is our prediction for the spectral index between 1.1 and 3.1 GHz.

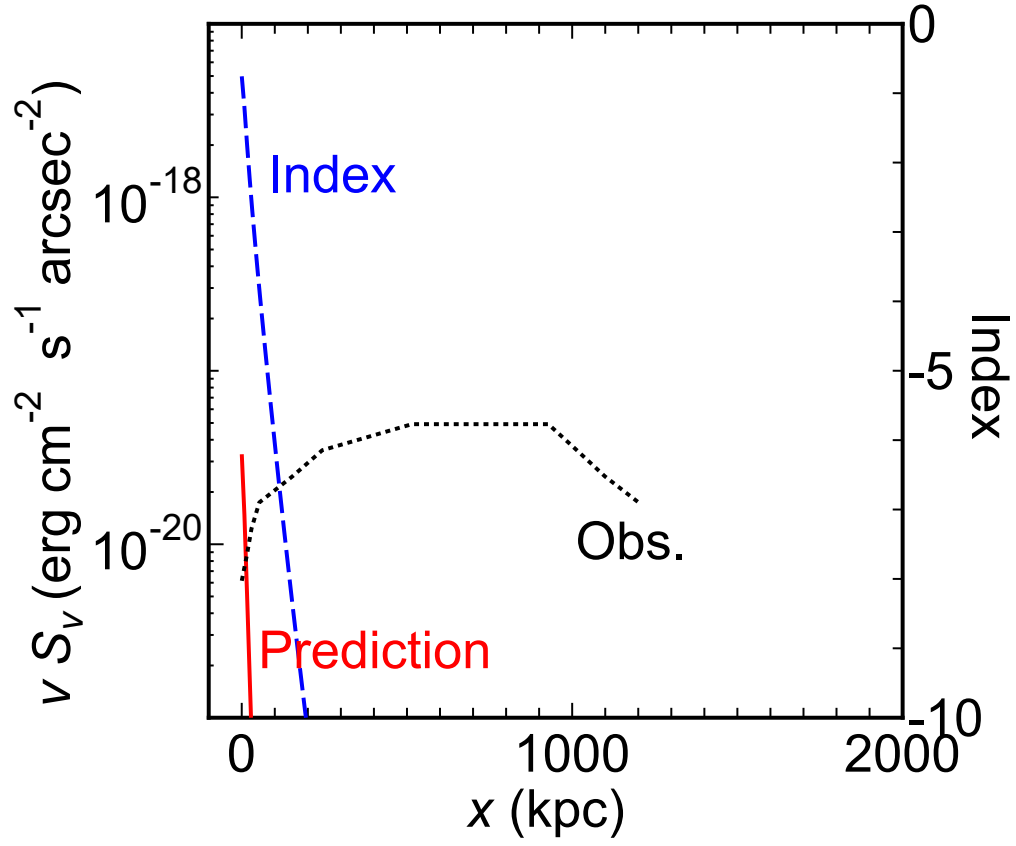


Fig. 4.— Same as Figure 3(c) but for  $\eta = 1$  and  $\chi_e = 3 \times 10^{-6}$ .